Low-Cost “Ball and Plate” Design and Implementation for Learning Control Systems

Deejay Stander
Department of Electrical and
Electronic Engineering
Universidad de los Andes
Bogotá, Colombia

Santiago Jiménez-Leudo
Department of Electrical and
Electronic Engineering
Universidad de los Andes
Bogotá, Colombia

Nicanor Quijano
Department of Electrical and
Electronic Engineering
Universidad de los Andes
Bogotá, Colombia

Abstract—Undergraduate engineering students require a practical application of theoretical concepts learned in classrooms in order to appropriate a complete management of them. Our aim is to assist students to learn control systems theory in an engineering context, through the design and implementation of a simple and low cost ball and plate plant. Students are able to apply mathematical and computational modelling tools, control systems design, and real-time software-hardware implementation while solving a position regulation problem. The whole project development is presented and may be assumed as a guide for replicate results or as a basis for a new design approach. In both cases, we end up in a tool available to implement and assess control strategies experimentally.

I. INTRODUCTION

Motivating students and increasing their comprehension will always be a challenge in learning processes. Experimental knowledge is fundamental in engineering, this is why laboratories where students can interact with real systems are so important. In [1], Dixon emphasises that laboratory experiences are particularly important and presents the basis that supports it with regard to the teaching of control systems. Often, mathematical models and simulations are used to satisfy their need of experimental knowledge, but a computational model not always resembles a real system behavior.

The course coordinators for the undergraduate class Control System’s Analysis at Universidad de Los Andes have designed laboratory sessions using expensive plants that require full supervision. This fact restricts the students potential to experiment because they have to be excessively careful with the expensive equipment.

Based on this, low cost plants have been proposed in order to meet the students needs, removing the restrictions previously discussed. Indeed, in the 90’s, Rockwell laboratories introduced the Ball and Plate (B&P) system as a challenge, then becoming one of the favorites for education in control engineering. By virtue of its flexibility, the plant can be used to test and validate numerous control strategies. The main objective of the plant is to control the position of a ball, tilting the plate on which it rests. Several approaches have been taken to implement a B&P system, where different selections have been made for each of the control elements. Here, we present a comparison between the proposals that stand out, assessing each of the aspects that are faced in each stage of the project development according to the low-cost simple replication context for learning control systems.

A. Mechanical design

The authors in [2], [3] and [4] present different mechanical designs for the system. In [2] an L-shaped mechanism is used to couple two motors to the plate and provide the necessary torque. This approach ignores the small angle plate deflections that the shifted rotation axis implies, which leads to the ball slipping on the plate. The mathematical model assumes the plate is rotating on its inner axis, which differs from the real arrangement. To deal with it, they could have aligned the motors axes with the plate.

A different approach is presented in [3] where the plate is round and the system has 3 actuators. This system is much more complex than the first one because the axes of rotation of the plate are not independent, and modelling the system mathematically becomes arduous; additionally, it has the same problem of a non-zero turning radius. The design of [4] is simpler than the first two and does not have the non-zero turn radius problem, but the mechanism is more elaborated as it employs long arms to transfer the torque from the motors to the plate. In order to prevent damaging the systems, the controller must react slow when both motors are moving.

An improvement on [4] is presented in [5], where the system employs an inner plate and an external frame, to decouple the two angles. The metal frame is turned by one motor that is fixed to the base of the system and the plate is driven by a motor fixed to the metal frame. Each degree of freedom is independently driven by each motor. The non-zero turn radius problem in [2], [3], [6] and [7] is solved in [4].

B. Ball sensing

Another challenge of designing the B&P system is to sense the state of the ball. The two most common approaches are based on an overhead camera ([3], [4], [5]) or a touch screen ([2], [7]) on the plate. Both approaches require instrumentation circuitry to process the information which represents additional costs. Another option is
to design an array of phototransistors that sense the ball by the interruption of light \[^{[6]}\], but this method is very cumbersome and unreliable.

C. Controller

With the ball’s state sensed and the plates turning successfully, the hardware is out of the way, and the system’s controller is to be considered. The B&P plant is a nonlinear multiple inputs, multiple outputs (MIMO) system which needs two controllers in a cascade layout to control the ball’s position. The first control loop drives the motors by means of varying the voltage. The outer loop controls the ball’s position by adjusting the plate’s angles. The plate must reach the angle given by the outer loop before the ball’s position is sensed again.

In \[^{[2]}\], \[^{[4]}\], \[^{[5]}\] and \[^{[6]}\] a PID controller is implemented for the inner loop with low stabilizing time. For the outer loop, a variety of strategies have been proposed and implemented. In \[^{[3]}\] a fuzzy logic controller is designed and simulated with good results, but ignoring the time it takes to measure the ball’s position. This issue will be significant because the system relies on a camera and real-time image processing.

In \[^{[4]}\] a fuzzy logic supervisor for a PD controller and a sliding mode controller with a fuzzy approximate are used. Both controllers perform acceptable, but each takes roughly 10 seconds to stabilize the ball’s position with high overshoot, due to the time it took to measure the ball’s position.

In \[^{[5]}\] a linear full-state tracker and a sliding mode controller are compared. Both controllers are designed for tracking a sinusoidal signal. The full-state tracker is precise but is slow to follow the signal, while the sliding mode controller is very quick. They found that by considering the system’s nonlinear terms, the controller can be much more precise and quick.

The main contribution of this paper is to present a B&P development description as a very low cost plant ($100 USD, i.e., much less than most control plants) that allows not only to implement control techniques and evaluate their performance but also to be replicated as a whole engineering project with all the stages that should be faced by an engineer in formation. The B&P plant also provides an opportunity for students to work without a high and expensive damage risk, as everything is easy and cheap to repair. Additionally it does not require high level of technical skills to be replicated, and the electronic components employed are commercially available at most online stores.

The remainder of the paper is organized as follows. Section 2 describes the plant that is designed for the project, Section 3 introduces some of the system’s applications, Section 4 describes the modeling process. Furthermore, Section 5 explains the control used and finally Section 6 concludes the paper.

II. The Ball and Plate Plant

Now, we present our proposal for the B&P design and implementation, based on the literature review presented, boosting the best features of each of them within the context of a basic control systems undergraduate course. Each step of the development of the project is accomplished as follows.

A. Design

1) Mechanical Design: The implemented B&P system follows the design introduced in \[^{[5]}\] but instead of using an all aluminium proposal, acrylic is employed to reduce costs. The design has two degrees of freedom. In Figure 1 the constructed plant is presented.

![Figure 1. Ball and plate plant.](coffeebrain wiki)

The base of the system is acrylic, while the structure that holds the plates is made out of aluminium frames. One of the frames holds the camera above the plates and the other one has a bracket to hold the motor that drives the axis. A complete and detailed guide to build the plant can be found at coffeebrain wiki.

2) Actuation Mechanism: To drive the plates, two standard MG995 inexpensive and easy to acquire servo motors with operating torque of 15 Kg-cm (much more than required) are employed. The motion from the servos is transferred to the plate’s axis through 3D printed gears. The servo that controls the outer plate’s angle is fixed to the aluminium frame, while the other, which inclines the inner plate, is fixed directly to the outer plate. This election simplifies the plant dynamics and reduces the forces supported directly by the actuators, which prevent
their damage.

3) Ball Sensing: The ball’s position is sensed with a standard personal computer webcam and processed with MATLAB®’s Image Acquisition™ and Image Processing™ toolboxes. This approach is much cheaper than a touch screen and is divided into the following steps:

- Taking a picture, and separating it into the three basic colors (RGB).
- Thresholding the image to isolate the basic components of the desired color and removing noise as is shown in top of Figure 2.
- Applying the Blob Analysis [8] to identify the position of the ball in the filtered image.

The framerate of the camera and the speed at which the computer can process the image determine the speed of the system. The implemented system’s camera could maintain a stable framerate of 25 pictures per second.

B. Controller

Two PID controllers are designed to control each of the ball’s position coordinates. Although non-linear control techniques could be designed and applied, the theoretical approach of the basic control course provides tools just for linearise the system and design classical strategies as a PID, IMC, root locus and state-feedback control. The PID’s are tuned based on a linear transfer function that is obtained using the System Identification Toolbox™ based on experimental data.

For controlling the servo-motors, an Arduino® board, that is connected through serial communication to Simulink® is employed. This is achieved using the Legacy MATLAB® and Simulink® support for Arduino® toolbox which let us control the outputs of the Arduino® board in real-time directly from Simulink®. A guide to using this toolbox can be found at the link previously provided. The scheme of the outer control configuration is shown in Figure 3, which allows to manually specify the set-point while the plant is running.

C. Simulation

First, the simulation for the entire system was developed using Simscape™ and Simulink®. A 3D model is employed in order to create an animation (Figure 4) that illustrates the plant state, which also allows to evaluate the virtual performance of different control strategies.

The simulation allows to evaluate the controller performance under certain circumstances without the risk of damaging an expensive physical plant. It is a perfect testbed for assessing control strategies designed by students as they can instantaneously realize the effect of changes.

III. MATHEMATICAL MODEL

To model the plant, a few assumption are made to simplify the math needed, these are:

- The ball never loses contact with the plate.
- There is no friction.
- The ball does not slip.

The parameters that will be used are defined in Table I.
model the system’s dynamics the Euler-Lagrange method is used. The general equation is

\[
\frac{d}{dt} \frac{\delta T}{\delta \dot{q}_i} - \frac{\delta V}{\delta q_i} = Q_i, \quad (1)
\]

which relates the kinetic energy \(T\), the potential energy \(V\), and the total force of the system \(Q\). The system has four degrees of freedom, two for the ball’s movement and two for the plate’s rotation. This means the system has four generalized \(q_i\) variables of motion. Here, \(x, y\) and \(\beta\) are the ball’s position variables, assuming \((x, y) = (0, 0)\) at the center of the plate. The plate’s inclination is expressed by means of \(\alpha\) and \(\beta\).

The kinetic energy of the ball consists of its rotational and linear components for each axis, as shown in Equation (2). The ball’s rotational velocities are \(\omega_x\) and \(\omega_y\), and the derivative of the its position is its linear velocity, \(v_x\) and \(v_y\).

\[
T_b = \frac{1}{2} m_b (v_x)^2 + \frac{1}{2} I_b (\omega_y)^2 + \frac{1}{2} I_b (\omega_y)^2 + \frac{1}{2} I_b \omega_x^2 \quad (2)
\]

By relating the ball’s rotational velocity with its linear velocity, Equation (2) can be simplified to Equation (3), i.e.,

\[
T_b = \frac{1}{2} \left[ m_b \left( \dot{x}_b^2 + \dot{y}_b^2 \right) + I_b \left( \dot{\omega}_x^2 + \dot{\omega}_y^2 \right) \right] = \frac{1}{2} \left( \frac{m_b + \frac{I_b}{r_b} \dot{x}_b - m_b \left( x_b \dot{\alpha} + y_b \dot{\beta} \right) \dot{\alpha}} \right. \left. + m_b \dot{\omega}_y \dot{\beta} \right) \dot{\beta} \quad (3)
\]

To model the plate’s dynamics, it is necessary to take into account the effect of the ball on the plate, by means of the parallel axis theorem, which relates the effect that the shifted axis of rotation of the ball has on the plate’s inertia. As a general equation, it states,

\[
I_{z,P} = I_{z,G} + mr^2,
\]

where \(z\) is the axis on which the effect from \(G\) to \(P\) is to be calculated. Here, \(m\) is the mass of the shifted element and \(r\) is the turning radius. Applying this theorem, and the plate’s rotational energy, the plate’s kinetic energy can be obtained as follows,

\[
T_p = \frac{1}{2} \left( I_p + I_b \right) \left( \dot{\alpha}^2 + \dot{\beta}^2 \right) + \frac{1}{2} m_b \left( x_b \dot{\alpha} + y_b \dot{\beta} \right)^2
\]

The system’s total kinetic energy is defined now as,

\[
T = T_b + T_p
\]

Finally, the ball’s potential energy is given by,

\[
V_b = m_b gh = m_b g (x_b \sin \alpha + y_b \sin \beta),
\]

where \(h\) is the relative attitude of the ball with respect to a zero reference attitude in the horizontal inclination of the plate. The system’s total energy is given by the Langrangian

\[
L = T_b + T_p - V_b.
\]

Calculating the partial derivatives of \(L\) and using Equation (1), the system’s nonlinear equations can be found as follows,

\[
\tau_x = \left( I_p + I_b \right) \ddot{\alpha} + m_b x_b \ddot{\alpha} + 2 m_b x_b \dot{x}_b \ddot{x}_b + m_b x_b \ddot{y}_b \ddot{\beta} - m_b g \cos \alpha \quad (8)
\]

\[
\tau_y = \left( I_p + I_b \right) \ddot{\beta} + m_b \ddot{y}_b \ddot{\beta} + 2 m_b y_b \ddot{y}_b \ddot{\beta} - m_b g \cos \alpha \quad (9)
\]

Equations (8) and (9) describe the system’s behavior when an external torque \(\tau_{x,y}\) is applied. Equations (10) and (11) relate the ball’s behavior with the plate’s inclination. Using the equation that models the ball’s behavior, ignoring the terms that include the plate’s angular velocity and evaluating the parameters in the values of the physical system, we can find the continuous transfer function for each axis of the ball’s position.

\[
P_x(s) = \frac{X_x(s)}{\ddot{\alpha}(s)} = \frac{g}{\dot{\alpha} s^2}, \quad (12)
\]

\[
P_y(s) = \frac{Y_y(s)}{\ddot{\beta}(s)} = \frac{g}{\dot{\beta} s^2}. \quad (13)
\]
IV. CONTROLLING THE PLANT

PID controllers are designed for the B&P plant, and tuned based on the model identified. To evaluate their performance, the same controller is applied to different models (the analytical model of the previous Section; an experimentally identified model; a Simscape-based model; and the physical plant) and the performance is compared. A first controller is tuned to reach a setpoint, the second one is tuned for tracking, and the last one has a perturbations rejection target.

For testing the set-point tuned controller, a 5cm step input is specified and their performance are measured. Table II summarizes the results of some performance criteria for each of them. In Figure 5 the step response of a single axis of the physical plant with the set-point tuned controller is presented.

<table>
<thead>
<tr>
<th>Set-point</th>
<th>Models</th>
<th>Rise T. [s]</th>
<th>Overshoot [%]</th>
<th>Settling T. [s]</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Model</td>
<td>3.64</td>
<td>2.1</td>
<td>2.36</td>
<td>6.365</td>
<td></td>
</tr>
<tr>
<td>Identified</td>
<td>3.2</td>
<td>4.2</td>
<td>2.36</td>
<td>10.32</td>
<td></td>
</tr>
<tr>
<td>Simscape</td>
<td>3.12</td>
<td>2.26</td>
<td>2.08</td>
<td>8.364</td>
<td></td>
</tr>
<tr>
<td>Phys. Plant</td>
<td>0.822</td>
<td>5.8</td>
<td>0.74</td>
<td>13.36</td>
<td></td>
</tr>
</tbody>
</table>

Table II

SET-POINT PID PERFORMANCE ASSESSMENT

Then, a sinusoidal reference is defined as input, applying the tracking controller. The physical ball follows the reference signal successfully, as shown in Figure 6 mainly because of the robustness of PID controllers and because of the the ball’s speed regulation of the derivative action. Due to the physical dynamics of the system, specifically due to the outer plate’s inertia, the X axis of the position tends to have more overshoot than the other axis.

In order to test the perturbations rejection controller we propose the following process. The ball is moved by hand to one of the corners of the plate and then the controller acts. Indeed, it successfully moves the ball back to the reference position without much overshoot, as is shown in Figure 7. The controller successfully stabilizes the ball’s position to the setpoint (0 in this case) in each axis.

All of these results may be compared between physical and computational models in order to recognize their validity and performance differences.

V. CONCLUSIONS AND FUTURE WORK

We present a complete Ball and Plate design and implementation procedure. The plant is easy to use and to replicate at a low cost, so ideally anyone can use this guide to obtain the same results that are presented in this paper. It also provides the student the experience of making physical systems, and exposes him to the engineering process of designing, building and testing a real system. The plant can be used by undergraduate students without risk of injury or damaging expensive components so they will be motivated to explore the plant behavior. Building the plant provides an understanding of physical considerations that must be taken into account during the control design process. This provides students with a unique opportunity to learn through experience about control system’s theory. The designed plant can be controlled with Simulink in real time, which allows to design and implement almost all kind of controllers on it. Additionally the models that were obtained can be used to test controllers before physical implementation, which provides the means to safely explore and test.

The comparisons presented in the previous chapter show a very important underlying quality of the system. The math model, the Simscape simulation, the identified transfer function and the real system can all be
stabilized by the same controller and all of them have a similar behavior. This fact shows the validity of all the representations and will let students understand why modelling is important and useful. In the same regard, when students compare the models to the real system they can better understand the effects of approximations and assumptions.

Future work for the project has two ramifications. The first is to design experiments using the plant to help students learn about specific topics. The second, is to use this project as a guide for students to develop other types of controllable plants, like an inverted pendulum or something more focused on industry processes like mixing and controlling concentration.

REFERENCES


